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Irregular Surface Mesh Topology for Volumetric Deformable Model

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Abstract

This paper explores the possibility of employing a surface model for volume simulation. The issues are the non-existence of internal volume that causes object collapse and the physical properties estimation of the model. Therefore, properties distribution scheme based on a mass spring system is proposed where values for mass and the inner support spring stiffness at the nodes are estimated based on the relationship of the surface nodes to the object centre. Local and global volume behaviours are preserved when the surface model is simulated under the influence of gravity, deformed by external forces and topologically refined. The proposed scheme contributes towards the employment of a deformable surface model for soft volume simulation with haptic interaction.

Keywords: Physical-based modelling, volume simulation, surface mesh, property estimation

1 Introduction

Figure 1. A breast surface model with irregular mesh topology

Soft volume simulation requires both the geometrical representation as well as the physical modelling of the object of interest. For instance, medical simulation requires the simulated organ to look and to behave as identical as possible to the real organ. To achieve this, a volumetric object is generally made up of tetrahedral elements that represent its surface and internal volume. However due to the complexity of a volumetric mesh and the high computational overhead it requires, there has been an increase in the use of a surface mesh to emulate volume behaviour. A surface mass spring system has been employed to simulate solid and cavernous objects such as blood vessels (Brown et al., 2001), stomachs (Choi et al., 2005) and muscles (Nedel and Thalmann, 1998; Hong et al., 2006). However, emulating volume behaviour is a great challenge due to the non-existence of the internal spring network to represent the internal volume. Most volume object such as soft tissue, which is essentially made up of water, is incompressible (Picinbono et al., 2001). So naturally, volume is preserved during deformation despite the elasticity behaviour. The level of difficulty is increased when an
irregular mesh topology makes up the surface. Irregular mesh topology refers to the non-uniform node concentrations throughout the surface of the object as shown in figure 1. Therefore, to emulate volume behaviour, the local and global behaviours of the object have to be preserved during simulation regardless of the mesh topology and the non-existence of internal volume.

2 Related Works

Mass Spring System (MSS) consists of point masses connected by elastic spring links. When the system is mapped against the geometric mesh as shown in figure 1, the masses are the mesh nodes and the springs are the edges. Properties such as mass values and stiffness values have to be distributed to the spring system. To simplify properties estimation, the object mesh usually assumes regular topology (Delingette, 1998; Gelder, 1998; Bourguignon and Cani, 2000; Brown et al. 2001). Deussen et al. (1995) modified the irregular mesh to produce more regular node concentrations. Therefore, uniform properties were assumed. However, the object surface should not necessarily have the same amount of node concentration. Irregular mesh topology or multi-resolution allows visual acuity as well as higher computation to be concentrated within specific areas on the surface. The system should effectively limit the computations to the portions of object that undergo significant deformations (Brown et al., 2001). The shape of the object determines which surface region requires a more refined topology than another. Furthermore, a surface mesh area within the radius of influence of the interaction force can be subdivided to achieve a more refined geometry (Zhang et al., 2002). Consequently, properties re-estimation is required. The existing regular re-estimation methods such as discussed in (Choi et al., 2005) preserved the local properties but did not preserve the deformation behaviour. The displacement patterns of a node within the coarse and refined area do not coincide.

However, manipulating the properties of the surface mesh alone will not guarantee the preservation of object volume during simulation as shown in figure 2.

To achieve volume preservation using a surface model, existing method converts the surface mesh into a volume mesh where internal spring network is created. This imposed high computational overhead due to the re-meshing process as well as the complexity of a volume mass spring system. This does not guarantee volume preservation as the resulted volume mesh is still based on 2 dimensional springs that represent the edges. Bourguignon and Cani (2001) introduced artificial springs to preserve the object volume based on the individual tetrahedral elements. The consequent object is stiffer than it should be (Hong et al., 2006). Attempts on muscle simulation (Nedel and Thalmann, 1998; Aubel and Thalmann, 2000) addressed shape preservation and not volume. A more effective solution is based on independent inner springs (figure 3) that preserve the shape of the object during simulation (Mendoza et al., 2002; Vassilev & Spanlang, 2002; Zhang et al., 2002; Laugier et al., 2003; Choi et al., 2005; Marchal et al., 2005; Maciel et al., 2005). The springs, which are embedded at the mass nodes, each has zero rest length. The method has successfully preserved the shape when the MSS reaches the
equilibrium stage. However, the influence of gravity is generally excluded and the spring stiffness is estimated based on some fine-tuning processes or assumed uniform. The stiffness estimation has not considered the material properties of the object.

Figure 3. Inner support springs

3 Proposed Scheme
The proposed scheme manipulates the use of the inner support springs to preserve the volume of the object during deformation as well as under the influence of gravity. In order to achieve this, mass and the stiffness of the springs have been estimated based on the topology of the surface mesh. The relationship of the surface nodes with the object centre (Vassilev and Spanlang, 2002) as well as with the neighbouring triangular elements of the mesh (Cignono et al., 1999; Villard and Borouchaki, 2002) has been explored to address the needs towards the estimation of the MSS properties.

3.1 Mass Estimation
Based on the relationship, the inner volume of the surface mesh can be artificially discretised into tetrahedral elements as shown in figure 4. These elements represent the volume under the triangular elements of the surface model.

Figure 4. Inner volume discretisation of a surface model with irregular mesh topology

Therefore, if the total mass of the object is $M$, mass at node $i$ is:

$$m_i = \sum_{t=1}^{n} C_i \frac{V_t}{V} M$$

where $V_t$ and $V$ are the volume of the tetrahedron under the surface triangle $t$ and the total object volume respectively. $C_i$ is the distribution coefficient at node $i$ in regards to the neighbouring triangles. The simplest approach is to consider the barycentric relationship of $i$ with the neighbouring triangles (Bourguignon and Cani, 2000; Bielser, 2003). Consequently, $C$ is 1/3. However, this is incorrect if the nodes are of different distances $L$ from the object centre. A
distance correction factor is required to estimate the distribution of the mass at the triangles to the member nodes. Therefore,

\[ C_i = \frac{L_i - L}{2L}, \quad i \in \mathcal{t} \]

where \( L_i \) is the total length of the nodes of triangle \( \mathcal{t} \) to the object centre. For, a triangle \( C_0 + C_1 + C_2 \) is equal to 1. This new coefficient addresses the triangle’s normal relative to the object centre.

### 3.2 Spring Stiffness

The surface spring stiffness is determined based on the irregular method proposed by Gelder (1998). The estimation for the inner spring stiffness has generally assumed a more regular mesh topology. However, the individual volumes of the artificial tetrahedrons that represent the surface triangles provide the correction factor that determines the spring stiffness at each node based on the material properties. The proposed scheme aims to explore the multi-dimensional aspect of the inner spring. In this paper, the springs assume isotropic behaviour. Hence, based on figure 4, spring stiffness \( K \) at node \( i \) is:

\[ K_i = E \sum_{m} \frac{V}{L_i^2} \]

where \( L_i \) is the distance of node \( i \) from the object centre and \( E \) is the elasticity factor (Young’s Modulus) of the inner volume.

### 4 Experimental Findings

The experiment framework has been implemented using Microsoft Visual C++, OpenGL and OpenHaptics. Phantom Desktop haptic device (figure 5) has been employed to provide the virtual interaction and the desktop PC has the specification of Intel Pentium 4, 2.40 GHz and 1 G RAM. The object mass is 50 g, surface elasticity (Young’s Modulus) is 1 N/M², inner elasticity factor is 10 N/M² and each time step denotes 0.01 s.

![Figure 5. Phantom Haptic instrument is used to interact with the virtual model](image)

Three schemes are compared in this experiment, which are:

- Scheme A: Regular estimation of properties
- Scheme B: Irregular mass but regular inner spring
- Scheme C: Irregular estimation as proposed in section 3
4.1 Properties Re-estimation

To illustrate the proposed scheme, a sphere model with irregular mesh topology has been employed which represents objects with a convex shape such as a human’s breast. To evaluate the estimation method, the local behaviour of the surface is observed. When a surface area is refined, properties will need to be re-estimated. The consequent refined area should behave identically to the original behaviour during deformation. Constant force is imposed on a node within the selected area. The displacement patterns of the same node within the coarse and refined area are plotted. If the patterns are identical, the deformation behaviour is preserved. This exercise is repeated with different force amounts. The displacement behaviours are studied, where 2 values are analysed:

i) The standard deviation between the 2 patterns determines the level of similarity between the patterns. The smaller the standard deviation, the more identical the patterns are,

ii) The mean deviation of the 2 patterns is calculated for the different schemes. The least value demonstrates that the node behaviour is preserved with the least deviation.

![Figure 6. The (a) standard deviation and (b) mean deviation of each scheme over force](image-url)
Based on figure 6, the proposed scheme C preserves the node behaviour within the coarse and refined area with the least standard deviation and mean deviation. This indicates that the properties are more correctly re-estimated after the mesh refinement. Not only that the patterns of the displacement are the most identical, but also produce the least deviation.

4.2 Volume Preservation
The current volume of the object is calculated during simulation based on the equation [4]. This equation supports both concave and convex surface due to deformation.

$$V = \frac{1}{3} \sum_{n} A(n(x_i + x_j + x_k) + (y_i + y_j + y_k) + (z_i + z_j + z_k))$$

where, volume $V$ is the accumulation of the volume based on the individual triangle nodes relative to its normal and $A$ is the area of the triangle $t$. The object is put under gravity and the visual observation concludes that the right amount of inner support has been estimated for the mass at the nodes. Figure 7 shows that Scheme C provides the correct visual representation compared to B.

![Figure 7. Gravity test: (a) Scheme B (b) Scheme C](image)

Based on the volume variation over time (figure 8), the variation percentage of the volume relative to the original volume is maintained at around 0.03%.

![Figure 8. Mean volume variation over time for Scheme C](image)
When a constant force is imposed on a node, runtime volume is calculated at each time step until the object reaches the state of equilibrium. Figure 9 shows that scheme C preserves volume with the least variation compared to A and B. This proves that the proposed scheme improves volume preservation during simulation.

![Figure 9. Mean volume variation over different force amounts](image)

5 Conclusions
The proposed scheme irregularly distributes mass and inner spring stiffness in regards to the surface mesh topology. This is a feasible approach towards preserving a constant volume during simulation even when gravity is switched on. The scheme improves properties re-estimation upon topological alterations such as surface mesh refinement, where the local properties and behaviour are preserved. The stiffness of the inner springs is derived based on the material property which dimensionality can be extended to address anisotropy. Currently, the proposed scheme assumes that the object has a convex shape with known centre. Further works include addressing the global deformation effect based on the orientation of the interaction force to create a more accurate global deformation effect despite of the non-existence of the internal volume.

6 References