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Content and Advertising: TV Media Competition in a Mixed-duopoly Market

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Abstract. This paper investigates the advertising intensity and channel content in a mixed-duopoly market. We find that there is less content difference in a mixed-duopoly market than in a private-duopoly market. The private channel is worse off when it competes against a state channel since the private channel faces more intense competition in content and viewing price. We also extend our analysis to social welfare investigation and policy implications. We show that the mixed-duopoly market is socially preferred to the private-duopoly market. It is also found that government intervention by using state channel only cannot reach social optimum.

JEL classification: D43, L13, L33, L82
Keywords: TV media, Mixed duopoly, Two-sided markets, Competition, Regulation

1. Introduction

The world has entered the digital TV era. Developed countries as well as developing countries have either completed or announced their transition from analogue TV to digital TV. For example, UK television broadcasters started to transmit digital signal in 2007 and will finish the switch-off by the end of 2011; in Canada the government has ruled that television stations would be forced to switch to ATSC digital broadcasting by August 2011; in the United States all full-power television will be digital by no later than February 2009, and analogue transmissions will be terminated; in China the switch-off is scheduled to be in 2015. The digital technological change is going to have critical and wide-ranging impacts on television broadcasting. In fact, with the adoption of digital encryption, TV channels may charge viewers directly, in addition to obtaining the funding from advertisers. Furthermore, the digital technology significantly increases the number of channels, due to the relaxing of spectrum constraints.
A large number of economic articles have argued that traditional broadcasting systems based on analogue transmission cannot bring the most efficient resource allocation to viewers, because of commercial media appealing to mass market and neglecting niche viewers. State broadcasters are willing to serve these markets as they have social benefits but are not covered by private broadcasters due to perceived lack of profitability, because state broadcasters have different aims from private broadcasters. Indeed, many countries have been using state broadcaster to improve efficiency in their national TV broadcasting markets, such as BBC in the UK, ABC in Australia, NHK in Japan, etc. Broadcasting markets all over the world are characterized by the coexistence of private channels and state channels. This is not a distinctive characteristic of TV media markets in developed countries; it also characterizes developing countries such as China and Middle-Eastern nations. According to Prat and Stromberg (2005), in the average Western European country about 50 percent of the five television channels with the largest audience are state-owned. This proportion increases to 70 percent in East Asia, 85 percent in Africa, and 94 percent in the Middle East. Since the nature of television broadcasting changes with the introduction of digital technology, it is important to re-investigate the competition between private television and state television.

In recent years, many debates have surrounded TV media competition in the digital world. One of the main concerns is what is the challenge to market competition and regulation in the TV broadcasting industry with the innovation of digital technology? Recent papers, such as Gabszewicz et al. (2004), Gal-Or and Dukes (2003), Anderson and Coate (2005), and Peitz and Valletti (2008) have used a Hotelling model to study two private media operators competing in both programming and advertising levels in a two-sided market. Armstrong and Weeds (2007a) analyze channel program quality competition by adopting a two-sided market model to study the program quality competition in both pay-TV and advertising-funded TV markets. However, these papers focus on the competition between private TV channels, where mixed-duopoly competition in the TV media market has not been investigated sufficiently. As mixed oligopoly is still the dominant market structure in the TV media industry, from a theoretical standpoint as well as policy standpoint, it makes sense to study TV channel competition based on a mixed-duopoly market. Furthermore, we expect that many of the conclusions from models of private markets do not carry over to the mixed-duopoly setting.

To my knowledge, mixed-duopoly competition in the TV media market is examined by relatively few papers. Kind et al. (2006) study price competition in the

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2 Steiner (1952) Beebe (1977) and Spence and Owen (1977) have discussed programme decisions in the traditional analogue TV market.

3 According to DCMS (2005), the government suggested to expand the BBC's mission with five distinctive purposes: 1) sustaining citizenship and civil society; 2) promoting education and learning; 3) stimulating creativity and cultural excellence; 4) representing the UK, its nations, regions and communities; 5) bring the UK to the world and the world to the UK. For further discussion of public broadcasting see Armstrong and Weeds (2007a).
TV media market by introducing a state channel which maximizes total social welfare as a competitor to a private channel, with an exogenous content difference. In contrast to their state channel's objective function, we assume that the state channel maximizes the total welfare of viewers with a break-even constraint. Indeed, it seems inappropriate to assume simply that state TV channels maximize total social welfare. For example, neither the state station (i.e., BBC) in the UK nor the state station (i.e., CCTV) in China takes the advertiser's welfare into account when they compete in their national TV broadcasting markets respectively. Armstrong and Weeds (2007b) look at programme quality competition between a commercial broadcaster and a public broadcaster, which maximizes viewer welfare overall under Pay-TV structure and free-to-view structure. In this paper, we focus on channel content competition between a private channel and a state channel.

With respect to the model settings, this paper is closely linked to and builds on Anderson and Coate (2005) and Peitz and Valletti (2008). Both papers put the competition of two platforms in a two-sided markets framework. Anderson and Coate (2005) use a Hotelling model to analyze whether too much or too little advertising is supplied in equilibrium for both free-to-air and pay-TV.\(^4\) Peitz and Valletti (2008) extend the discussion to endogenous content competition. They show that there is less differentiated content in a free-to-air market, but in a pay-TV market there is always maximally differentiated channel content. In a pay-TV market without program difference, platforms will compete profits down to zero in a Bertrand-style game. If a platform differentiates its program by moving away from the other it can make a positive profit. But in a free-to-air market, in order to attract advertisers, TV platforms compete aggressively by moving towards the rival.

In contrast, by introducing a state media platform, we can achieve different equilibrium results on pricing and content. Therefore, our paper offers an interesting comparison to Peitz and Valletti (2008). We focus on viewing price, advertising space, profits of channels and content provision between different types of channels. Our finding that there is less content difference in a mixed-duopoly market is in contrast to the Peitz and Valletti's (2008) finding that in a pay-TV framework with two private channels the channel contents are always maximally differentiated. The private channel is worse off when it competes against a state channel since the private channel faces more aggressive competition in content and viewing price. We also extend our analysis to a social welfare investigation and policy implications. Our results offer support to the opinion that a state channel can improve social

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\(^4\) They find that the equilibrium advertising level depends on the nuisance cost to viewers, the substitutability of programs, and the expected benefits to advertising. In the case of free-to-air, the welfare outcome is ambiguous: advertising is underprovided because the benefits of programming to viewers are high, relative to the benefits that advertisers get from contacting viewers; advertising is overprovided when program benefits are low relative to advertising benefits and nuisance costs are low. In the pay-TV situation, advertising may be underprovided, since the broadcasters compete for viewers by lowering their advertising levels, i.e., there is a business-stealing effect.
efficiency in that the mixed-duopoly structure is socially preferred to the private-
duopoly structure. We also find that government intervention through only using a 
state channel cannot lead to the social optimum.

The structure of this paper is the following: section 2 outlines the basic 
model, section 3 analyzes channel competition in a mixed- duopoly market, section 4
discusses channel competition in a private-duopoly market, section 5 gives a 
comparison between the private-duopoly market and the mixed-duopoly market, 
section 6 extends the analysis to the welfare and policy implications, and section 7 
provides a conclusion to this paper.

2. The Model
Our model is a canonical two-sided markets model. In model setting, it is close to 
Peitz and Valletti's (2008) model with the exception that we introduce a state 
channel. In addition, the state channel maximizes the total welfare of viewers with a 
break-even constraint.

2.1 Platforms (TV channels)
There are two TV channels, indexed 1 and 2. Channel 1 is a private channel, and 
channel 2 is a state channel which maximizes the total welfare of viewers with a 
break-even constraint. Channels determine their channel contents by choosing 
locations on a standard Hotelling model. Channel 1’s content is  and channel 2’s 
content is 1−d2. Without loss of generality, we assume 0≤d1≤1−d2≤1.5 Content 
can be in the interval [0,1]. Viewers have to pay s_i for viewing (pay-per-view), and 
advertisers have to pay to put an advertisement on platform i. Channels have zero 
production cost. Hence the channel's profit is:

\[ \pi_i = b_i s_i + a_i r_i \]

where b_i is the viewership and r_i is the advertising price of channel i.

2.2 Viewers (Consumers)
Viewers of mass of 1 are uniformly distributed on the content interval [0,1]. Each 
consumer’s location represents his favorite type of channel content. For viewer x, if 
he chooses platform 1, the transportation cost \( t(x−d_1)^2 \) incurs a disutility, where 
t > 0; if he chooses platform 2, the transportation cost is \( t(1−d_2−x)^2 \). Advertising is 
purely informative, but the viewers dislike advertisements.6 The nuisance cost to 
watch an advertisement is \( \delta \).

We assume that there is single-homing on the viewer side. All the viewers 
watch TV and have the same intrinsic utility \( v \) of viewing. Hence, the indifferent

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5 Given this constraint, there is a unique pure strategy equilibrium in stage 1.
6 For an in-depth overview on the different views on advertising, please see the paper of Bagwell (2003).
viewer between platform 1 and 2 is $b_i$, where:
\[
v - \delta a_i - s_i - t(b_i - d_i)^2 = v - \delta a_z - s_z - t(1 - d_z - b_i)^2
\]
\[
b_i = \frac{1 + d_i - d_z}{2} + \frac{\delta}{2t(1 - d_i - d_z)}(a_z - a_i) + \frac{1}{2t(1 - d_i - d_z)}(s_z - s_i)
\]

2.3 Advertisers (Producers)
Advertisers are also producers. Advertisers of mass 1 sell products to viewers who are also the consumers of the products that are advertised. The producers have a zero marginal cost. If a product is produced at quality $\theta$, then consumers are assumed willing to pay $\theta$ for this product. Each producer is a monopoly in her industry and is assumed to extract the full surplus from consumers. This means that a product with quality $\theta$ is sold at price $\theta$. Producers differ with respect to the quality of the good which they offer. The distribution of quality is uniform on the interval $[0,1]$. Advertisers can advertise on none, one, or more channels. Channels are the same (ex ante) to all advertisers.

The profit of advertiser with type $\theta$ is:
\[
\pi = \theta b_i - r_i
\]

The marginal advertiser on channel $i$, is $\theta = \frac{r_i}{b_i}$. Hence, channel $i$’s advertisement revenue on per viewer is $R(a_i) = (1 - a_i) a_i$.

2.4 The Timing of the Game
TV channels, viewers, and advertisers play a three-stage game.

Stage 1: Channels choose their channel contents (locations on the Hotelling line) simultaneously.

Stage 2: Channels determine their viewing prices and advertising spaces simultaneously.

Stage 3: Advertisers and viewers choose channels.

We characterize the subgame perfect Nash equilibrium. In addition, we assume that the nuisance cost of advertisement is moderate, $(0 < \delta < 1)$.

3. Mixed-duopoly Competition
In this section, we analyze competition between the private channel and the state channel in viewing price, advertising space, and channel content. In stage 3, the

---

7 Anderson and Coate (2005) and Peitz and Valletti (2008) both give a detailed discussion on how the nuisance cost, $\delta$, affects the equilibrium on advertiser side. They point out that if advertising is very annoying, platforms would price it out of the market. This is clearly not our case of interest as the state channel in our model maximizes the total welfare of viewers with a break-even constraint, and so we rule it out.
viewership of channel 1 and channel 2 are $b_i = h_i$ and $b_i = 1 - h_i$ respectively. The number of advertisements is $a_i = 1 - \frac{h_i}{b_i}$. Rationally anticipating both the viewers' and advertisers' actions in stage 3, TV channels determine their viewing prices and advertising spaces in stage 2.

3.1. Equilibrium in Stage 2

The private channel (channel 1) maximizes its total profit:

$$\text{Max}_{s_1, a_1} \pi_1 = b_is_1 + a_ir_1 = b_1[R(a_1) + s_1]$$

The two first-order conditions are:

$$\frac{\partial \pi_1}{\partial s_1} = b_1 + \left[ -\frac{1}{2t(1-d_1-d_2)} \right] \left[ (1-a_1)a_i + s_1 \right] = 0 \quad (1)$$

$$\frac{\partial \pi_1}{\partial a_i} = (1-2a_1)b_1 + \left[ -\frac{\delta}{2t(1-d_1-d_2)} \right] \left[ (1-a_1)a_i + s_1 \right] = 0 \quad (2)$$

As advertisers have to use media platforms to reach viewers, the private channel sets its advertising space to maximize the joint welfare of the platform and its viewers. Then, the viewing fee $s_1$ is used by the channel to extract part of the surplus from its viewers. Combining equation (1) and (2), we find that $a_i = \frac{1-\delta}{2}$ is a dominant strategy for the private channel, which is independent of content, its own viewing price, and the strategy of the rival.

Substituting $a_i = \frac{1-\delta}{2}$ into function (1), we obtain:

$$s_1 = 2t(1-d_1-d_2)b_1 - \frac{1-\delta^2}{4}$$

In contrast, the state channel (channel 2) maximizes the total welfare of viewers with a zero-profit constraint:
viewer welfare of channel 1
\[ \max_{s_1, a_1} W_1 = [(v - b_1 s_1) b_1 - \int_{0}^{b_1} t(x - d_1)^2 \, dx] \]
viewer welfare of channel 2
\[ + [(v - b_2 s_2) b_2 - \int_{0}^{b_2} t(x - d_2)^2 \, dx] \]
subject to \[ \pi_2 = b_2 [(1 - a_2) a_2 + s_2] = 0 \]

Since only the viewers' welfare is considered, the welfare of advertisers is neglected by the state channel. The state channel chooses the same advertising space as the private channel to maximize the joint surplus of the platform and its viewers, and then passes all the surplus to its viewers by a negative viewing price, given its zero-profit constraint. Therefore, the viewing price and advertising space are:
\[ a_2 = \frac{1 - \delta}{2}, \quad s_2 = -\left( \frac{1 - \delta^2}{4} \right) \]
Combining the solutions of \( a_1, a_2 \) and \( s_2 \), we can obtain the solution of \( s_1 \), which is:
\[ s_1 = \frac{(1 - d_2 + d_1)(1 - d_2 - d_1)t}{2} - \frac{1 - \delta^2}{4} \]
The above analysis is summarized by the following Proposition:

**Proposition 1:** In stage 2, the state channel's viewing price and advertising space are \( s_2 = -\left( \frac{1 - \delta^2}{4} \right) \) and \( a_2 = \frac{1 - \delta}{2} \) respectively; the private channel's viewing price and advertising space are \( s_1 = \frac{(1 - d_2 + d_1)(1 - d_2 - d_1)t}{2} - \frac{1 - \delta^2}{4} \) and \( a_1 = \frac{1 - \delta}{2} \) respectively.

Because the state channel aims to maximize the total welfare of viewers, it does not have an incentive to charge a positive price to its viewers at all.\(^8\) In contrast, the private channel has an incentive to extract as much surplus from its viewers as possible, the extent to which depends on the competition on the viewer side. The higher the \( t \), the higher market power the private channel has; that is, the private

---

\(^8\) We do not model any production cost of channels in this paper. Also, we allow channels to subsidize viewers. Adding a cost to channels will not change our results significantly, as in a two-sided market model platform may subsidize one side if this side exerts a higher positive network effect to the other side. (See the detailed discussion in Armstrong (2006) and Rochet and Tirole (2004)).
channel may obtain a larger profit from its viewers.

3.2 Equilibrium in Stage 1

In this section, we analyze how TV channels choose their channel contents. Substituting the equilibrium strategies in Proposition 1 into the channel's expression for viewership in the first stage, we have:

\[ b_1 = \frac{1 - d_2 + d_1}{4}; \quad b_2 = \frac{3 + d_2 - d_1}{4} \]

For the private channel, its profit function simplifies to the following when Proposition 1 is incorporated:

\[ \pi_1 = \frac{t}{8} (1 + d_1 - d_2)(1 + d_1 - d_2)(1 - d_2 - d_1) \]

where \( \pi_1 \) is single peaked between \(-1-d_2\) and \(1-d_2\) with respect to \(d_i\). The first-order condition is

\[ \frac{\partial \pi_1}{\partial d_i} = \frac{t}{8} (1 - d_2 + d_i)(\frac{1-d_2}{3} - d_i) = 0 \]

Since channel content cannot be negative, therefore there is a unique solution for \(d_i\):

\[ d_i = \frac{1 - d_2}{3} \quad (3) \]

For the state channel, it maximizes the total welfare of viewers with a zero profit constraint. We rewrite the expression of \(W_r\) as:

\[
\begin{align*}
\text{Max} W_r = &\left[ v + \frac{(1 - \delta)^2}{4} \right] - \frac{(1 - d_2 + d_i(1 - d_2 - d_i)t b_i)}{2} \\
&\text{Total disutility from watching less preferred content} \\
&- \left[ \int_0^{b_1} t(x - d_i)^2 dx + \int_0^{b_2} t(x - d_2)^2 dx \right]
\end{align*}
\]
The second term of $W_r$ is the surplus of the private channel; the third term is the total disutility from viewing less-preferred content on the viewer side. The state channel faces a trade-off when it determine its content: on one hand, it has an incentive to move close to the private channel to squeeze the profit of the private channel so as to increase the surplus of the viewers on the private channel; on the other hand, the state channel needs to consider the utility loss from content polarization, if it moves too close to its rival. The closer to the extreme point on the left side of the content interval that the state channel moves, the worse the viewers who locate close to the extreme point on the right side become. Maximizing $W_r$ is equivalent to minimize the following:

$$
\Gamma = \frac{(1-d_2+d_1)(1-d_2-d_1)}{2}t + \int_{0}^{h_1}t(x-d_1)^2\,dx + \int_{0}^{h_2}t(x-d_2)^2\,dx
$$

Since function $\Gamma$ is smooth and continuous on $d_z$, we can characterize the state channel's best response of $d_z$, given that $d_i$ is fixed. Combining the best response functions of the state channel and the private channel in stage 2, we can give the following Proposition.

**Proposition 2:** There exists a unique equilibrium in the mixed-duopoly competition.

- In stage 1, the state channel's content decision is $d_1 = \frac{3\sqrt{3} - 7}{2} \approx 0.18$;
   - the private channel's is $d_i = \frac{3\sqrt{3} - 7}{2} \approx 0.18$;

- In stage 2, both channels have the same advertising space: $a_i = a_z = \frac{1-\gamma}{2}$; however, the viewing prices are different ($s_1 > s_2$): $s_1 = \left(3 - \sqrt{7}\right)^2 t - \frac{(1-\gamma)^2}{4}$, $s_2 = -\frac{(1-\gamma)^2}{4}$;

- In stage 3, both channels have different market shares on the viewer side: $h_i = \frac{3\sqrt{7} - 7}{6} \approx 0.16$ and $h_z = \frac{13 - 3\sqrt{7}}{6} \approx 0.84$.

**Proof:** See Appendix, pp. 151-153.
4. Private Duopoly Competition
In this section, we look into the competition in a private-duopoly market, where two symmetric private channels compete in viewing price, advertising space and channel content. This will then be compared to our results from section 3 in our discussion. We start our analysis from stage 2.

4.1 Competition in Stage 2
Given the advertising space and viewership of each channel, both private channels maximize their profits with respect to the viewing price \( s_i \) and the advertising space \( a_i \).

The equilibrium at stage 2 is characterized by the system of four first-order conditions

\[
\frac{\partial \pi_i}{\partial s_i} = b_i + \left( -\frac{1}{2t(1-d_i-d_j)} \right) \left[ (1-a_i)a_i + s_i \right] = 0 \quad (5)
\]

\[
\frac{\partial \pi_i}{\partial a_i} = (1-2a_i)b_i + \left( -\frac{\delta}{2t(1-d_i-d_j)} \right) \left[ (1-a_i)a_i + s_i \right] = 0 \quad (6)
\]

Solving the four conditions, we can have the following Proposition (for \( i = 1,2 \))

**Proposition 3:** In the private-duopoly market, there is a unique symmetric equilibrium on viewing price and advertising space:

\[
s_i = \frac{t(3 + d_i - d_j)(1-d_i-d_j)}{3} - \frac{1-\delta^2}{4} \quad (i = 1,2); \quad a_i = \frac{1-\delta}{2} \quad (i = 1,2).
\]

4.2 Competition in Stage 1
Substituting the equilibrium strategy in stage 2 into the TV channels' profit function, we rewrite the profit function as:

\[
\pi_i = \frac{t}{18} \left( 1-d_i-d_j \right) \left( 3 + d_i - d_j \right)^2
\]

The competition in this stage is the same as the standard Hotelling model with quadratic transportation costs, and both players locate at the extreme point, i.e., \( d_i = d_j = 0 \) (D'Aspremont et al., 1979).

Substituting \( d_i = d_j = 0 \) into the equilibrium strategies in stage 2 and stage 3,
we obtain the following Proposition:

**Proposition 4:** In the private-duopoly market, there exists a unique symmetric equilibrium.

- In stage 1, the channel content is maximally differentiated \((d_1 = d_2 = 0)\).

- In stage 2, the channels set the same viewer prices and advertising spaces:

  \[ s_1 = s_2 = t - \frac{1 - \delta^2}{4}; \quad a_1 = a_2 = \frac{1 - \delta}{2}. \]

- In stage 3, the channels have the same market share on the viewer side:

  \[ b_1 = b_2 = \frac{1}{2}. \]

5. A Comparison between Private-Duopoly Market and Mixed-Duopoly Market

First, we look into the strategy difference between the private channel and the state channel in the mixed-duopoly market in stage 2. Although the private channel and the state channel have the same number of advertisements as each other, the viewing prices are quite different. In order to maximize the total welfare of viewers, it is better for the state channel with a zero-profit constraint to subsidize its viewers. In other words, the state channel does not have any incentives to use market power to charge its viewers, as a positive viewing price decreases the welfare of viewers directly, given the content decisions in stage 1. In contrast, the private channel has an incentive to extract as much surplus from its viewers as possible, which determines that the private channel charges a much higher price than the state channel. However, if the taste difference on the viewer side is small \((\gamma < \frac{1 - \gamma^2}{4(3 - \sqrt{7})})\), it is also possible that the private channel subsidizes its viewers as well. All its revenue comes from the advertiser side. Instead, if the taste difference is quite big \((\gamma > \frac{1 - \gamma^2}{4(3 - \sqrt{7})})\), the private channel charges its viewers a positive price. It means that the private channel accumulates its profit from the viewers and the advertisers.

In the private-duopoly market, the parameter \(t\) affects the viewing prices of both channels. Indeed, \(t\) reflects the market power of the channels on the viewer side, which softens the price competition between the two private channels. Both private channels have the ability to extract some surplus from the viewers because they are close to local monopolists when they locate at the extreme points of the Hotelling line. In other words, given the number of advertisements on each channel, they can increase their viewing fees slightly, without losing their viewers, because of the low
price elasticity on the viewer side. But when one of the private channels is replaced by a state channel, the other private channel faces a very aggressive viewing price from the state channel. Therefore the existing private channel has to charge a much lower price to its viewers than when it competes against a private channel.

Second, we compare the equilibrium strategies in stage 1 between the private-duopoly market and the mixed-duopoly market. The content decisions in the mixed-duopoly market is striking. The channel contents are neither maximally differentiated nor identical between the private channel and the state channel. As we have seen, in the private-duopoly market the contents are maximally differentiated between two channels (Peitz and Valletti 2008). The private channels have incentives to move far away from each other to increase their market power to extract more surplus from the viewers. However, in the mixed-duopoly market the channel contents are less differentiated, compared with the private-duopoly market. The state channel does not have an incentive to locate far away from the private channel in order to reduce the surplus of the private channel. But the state channel does not want to move too close to the private channel, since it knows that the private channel has an incentive to move away to avoid intense price competition. If they both move close to the extreme point on the content interval, this will result in a decrease in the total welfare of the viewers, even though the welfare of the viewers of the private channel is improved. This content duplication will not happen in the mixed-duopoly market.

Finally, from the point of view of the private channel, it prefers to compete in a private-duopoly market, rather than in a mixed-duopoly market. The competition result is stated in the following Proposition:

**Proposition 5:** The private channel is better off when it competes against a private channel rather than against a state channel.

**Proof:** See Appendix, pp. 151-153.

The private channel is strictly better off in the private-duopoly market because it can charge a much higher viewing price and obtain a much larger market share on the viewer side. When the private channel faces a state channel with a zero-profit constraint, it faces a very aggressive competition in the viewer market as the state channel values the welfare of viewers exclusively. The private channel has to compete aggressively as well in order to offer value to advertisers and so to earn advertising revenue. Therefore, its profit is much lower in the mixed-duopoly market than in the private-duopoly market. In addition, the private channel never has an incentive to transfer all the surplus obtained from advertisers to its viewers because of its objective (maximizing profit). Therefore, in the equilibrium the majority of

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10 In a traditional Hotelling model, two counteracting forces affect the location choice of each player. On the one hand, the player wishes to move close to his competitor to increase his "captive" customers who prefer his product. On the other hand, the player wishes to move far away from his rival in order to reduce price competition. In the private-duopoly market, the price competition effect is dominant.
viewers go to the state channel. The advertiser side cares about the number of viewers on each channel. With a smaller viewership, the private channel's profit on the advertiser side also reduces. Consequently, the private channel obtains a much lower profit than when it competes against a private channel.

6. Welfare and Policy Implication

In this part, we now examine which of the two market structures leads to a higher welfare. In order to answer this, we need to compare the total social welfare under both market structures, i.e., the private duopoly and the mixed duopoly.

In these two markets, the total welfare on the viewer side is \( V \), which is the sum of the individual utility from watching TV (as all watch TV by assumption). Also, the welfare of the advertiser side is equal in these two markets. In addition, the total nuisance cost to watching TV is the same in both cases, because of the same number of advertisements on TV channels. Therefore, we only need to compare the total disutility from watching less-preferred contents between the private duopoly and the mixed duopoly.

The disutility in the private-duopoly market is:

\[
\int_0^1 t(x - 0)^2 \, dx + \int_0^1 t(x - 0)^2 \, dx = \frac{1}{12} t \approx 0.083 t
\]

The disutility in the mixed-duopoly market is:

\[
\int_0^{3 - \sqrt{7}} t(x - \frac{3 - \sqrt{7}}{2})^2 \, dx + \int_{3 - \sqrt{7}}^{3} t(x - \frac{3}{2} (3 - \sqrt{7}))^2 \, dx \approx 0.054 t
\]

It is obvious that \( 0.083 t > 0.054 t \). We thus state without proof the following result.

**Proposition 6:** The mixed-duopoly market is relatively more efficient than the private-duopoly market.

This result offers support to the opinion that a state channel can improve social efficiency in that the mixed-duopoly structure is socially preferred to the private-duopoly structure. The appearance of a state channel changes the content decision significantly on the TV media market. With a state channel having our assumed objective function, the problem of over-differentiation of channel content has been solved, and the total welfare of viewer increases greatly due to the profit decline of the private channel and the decrease in disutility from watching less-favorable content.

We also investigate whether or not the social optimum can be produced. Peitz and Valletti (2008) study the socially optimal case with the same framework as ours. Hence, we simply state their results. They find that the optimal contents of the channels are \( d_1 = d_2 = \frac{1}{4} \); the optimal number of advertisements on each channel is...
$a_1 = a_2 = (1 - \delta)$. Compared with the socially optimal case, both the private duopoly and mixed duopoly in our model produces fewer advertisements. This can be explained by noting that neither the private channel nor the state channel takes the welfare of the advertiser side into account when competing against each other in the game. Unless viewers have been brought on board successfully, the private channel cannot charge its advertisers. This means that the private channel will firstly maximize the joint surplus of its viewer and its own by advertising space, and then it uses a viewing price to extract viewer surplus. It is straightforward to understand the state channel's decision on advertising space, as it only cares about the welfare of the viewer side. Advertising is annoying to the viewers, and so the social benefit of advertisement will not be fully considered by the state channel.

The utility loss from watching less preferred content is reduced in the mixed-duopoly case. However, it is still higher than the social optimal level. Although the private channel will locate at $d_1 = \frac{1}{4}$ if the state channel chooses $(1 - d_2) = \frac{3}{4}$, the state channel can improve the total welfare of viewers by moving a bit closer to the private channel. Some of the viewers become worse off, but the other viewers may be better off because the state channel can improve viewer welfare by squeezing the private channel's profit. The surplus that can be squeezed from the private channel is bigger than the net utility loss of viewers from watching less preferred contents. Therefore, the state channel has an incentive to move closer to the private channel. But this could cause another type of social inefficiency that channel contents are under-differentiated. We thus summarize the following proposition:

**Proposition 7:** There is underprovision of advertisements under both market structures. TV contents are overdifferentiated in the private-duopoly market; but TV contents are underdifferentiated in the mixed-duopoly market. Therefore, government intervention by use of a state channel only cannot reach the social optimum.

From the policy viewpoint, channel contents being underdifferentiated is not good news either. This result implies that it may be not enough to only use a state channel to intervene in the TV broadcasting market. Peitz and Valletti (2008) consider a scenario under which there does not exist a public-service broadcaster, and public-service broadcasting (PSB) obligations are assumed to have not been imposed on the private TV broadcasters. In other words, they investigate the theoretical limits to a non-PSB environment and aim to answer the question, “what would the market have done naturally had there not been obligations and had the PSB (like BBC) never existed?” However, in order to judge the implications of retaining PSB against other measurable alternatives, we need to bring a PSB into our discussion. Our model provides some support for retaining a PSB in the TV broadcasting market since the mixed-duopoly market is socially preferred to the private-duopoly market. Although we do not model public funding or government
grants to a PSB, it seems naturally a first step to investigate the implication of PSB in the digital TV age.

7. Conclusion
This paper has investigated the competition between TV channels in a mixed-duopoly market. By comparison with the competition in a private-duopoly market, we find that private channel is worse off when competing against a state channel than against a private one. In the private-duopoly market, channels have an incentive to maximally differentiate their channel contents in order to obtain a high profit. On the other hand, in the mixed-duopoly market, although the private channel has an incentive to move far away from the state channel, the state channel wishes to move closer to the private channel. This leads to a less-content differentiation than in the private-duopoly market. From the point of view of social welfare, the mixed-duopoly market has a higher efficiency than the private-duopoly market. But neither of them produce a socially optimal level of advertising space and channel contents.

This discussion contributes to the current debate on the new TV media regulation in the digital age. Peitz and Valletti (2008) and Anderson and Coate (2005) point out that the TV media market alone would not achieve the social optimal level of advertisement and content in the digital world. Our model further extends their discussion on TV media competition by introducing a state channel. In this paper, we have shown that the introduction of a state channel does not guarantee an optimal result either in the digital age.

Finally, we need to point out that we do not model the source of public funding or government grants to a state TV broadcaster. We consider a situation where state TV broadcasters have the same funding resources as commercial TV broadcasters, but with a different objective function. Therefore, given the intervention regime in our model, other types of regulation is needed to achieve a more efficient public broadcasting market.

8. Appendix
Proof of Proposition 2
The state channel needs to minimize $\Gamma$ with respect to $d_2$:

$$\min_{d_2} \Gamma = \frac{(1-d_2+d_1)(1-d_2-d_1)^2}{2}\int_0^1 h_t(x-d_2)^2 dx + \int_0^1 h_t(x-d_2)^2 dx$$

Integrating the second and third part, we have:

$$\min_{d_2} \Gamma = \frac{t}{8}(1-d_2+d_1)^2(1-d_2-d_1)^2 + \frac{t}{3}[h_t^2+(h_t-d_2)^2+(h_t-d_2)^2+d_2^2]$$

First, we derive the necessary condition for a local minimum. To make the
calculation easier, we calculate the first order derivative of $\Gamma$ with respect to $(1-d_2)$:

$$
\frac{\partial \Gamma}{\partial (1-d_2)} = \frac{t}{8} \left[ 3(1-d_2)^2 + 2d_1(1-d_2) - d_2^2 \right] 
+ \frac{t}{3} \left[ \frac{3}{4} \left( \frac{1-d_2}{4} - \frac{3}{4} d_1 \right)^2 + \frac{9}{4} \left( \frac{3(1-d_2)}{4} - \frac{d_1}{4} \right)^2 - 3(1-(1-d_2))^2 \right]
$$

$$
\frac{\partial \Gamma}{\partial (1-d_2)}
$$

can be simplified as:

$$
\frac{\partial \Gamma}{\partial (1-d_2)} = -12(1-d_2)^2 + (128-8d_1)(1-d_2) - \left(64-4d_1^2\right)
$$

There are two solutions for $(1-d_i)$, given \(\frac{\partial \Gamma}{\partial (1-d_2)} = 0\). However, considering the assumption \(0 \leq d_i \leq 1-d_2 \leq 1\), only the following solution is satisfied:

$$
(1-d_2) = \frac{(16-d_i) - 2\sqrt{(4-d_1)^2 + 36}}{3}
$$

(4)

The second-order derivative of $\Gamma$ with respect to $(1-d_2)$ is:

$$
\frac{\partial^2 \Gamma}{\partial (1-d_2)^2} = -24(1-d_2) + (128-8d_1) = 104 + 24d_2 - 8d_1
$$

This is positive definite on the interval \(0 \leq d_i \leq 1-d_2 \leq 1\). Therefore, we can confirm that function (4) gives the best response of the state channel on $d_2$.

From function (3) and (4), we obtain:

$$
d_i = \frac{3 - \sqrt{7}}{2} \approx 0.18
$$

$$
1-d_2 = \frac{3}{2} \left(3 - \sqrt{7}\right) \approx 0.54
$$

Replacing the solutions of $d_i$ and $(1-d_2)$ into the equilibrium results in stage
2 and stage 3, we have:

\[
 s_i = (3 - \sqrt{7})^2 t - \frac{1 - \gamma^2}{4} \\
 b_i = \frac{3\sqrt{7} - 7}{6} \quad \text{and} \quad b_2 = \frac{13 - 3\sqrt{7}}{6}.
\]

**Proof of Proposition 5:**

The private channel's profit in the private-duopoly market is:

\[
 \pi_i = b_i \left( s_i + R(a_i) \right)
\]

Inserting the equilibrium strategies in Proposition 2 into the above profit function, we have:

\[
 \pi_1 = \frac{1}{2} \left( t - \frac{1 - \gamma^2}{4} \right) + \frac{1 - \gamma}{2} \left( 1 + \gamma \right) = \frac{t}{2} = 0.5t
\]

In the mixed-duopoly market, inserting the equilibrium strategies in Proposition 3 into the profit function, the private channel's profit is:

\[
 \bar{\pi}_1 = \frac{3\sqrt{7} - 7}{6} \left( (3 - \sqrt{7})^2 t \right) = 0.019t
\]

Comparing \( \bar{\pi}_1 \) and \( \pi_i \), it is obvious that: \( \pi_i > \bar{\pi}_1 \)

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9. References


